

More Corrections to the Higgs Mass in Supersymmetry

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Abstract

In supersymmetry, the Higgs quartic couplings is given by the sum in quadrature of the weak gauge couplings. This leads to the prediction of a light Higgs boson, which still holds when considering loop corrections from soft supersymmetry breaking. However, another source of corrections, which explicitly depends on the scale of the mediation of supersymmetry breaking, is from generic hard breaking terms. We show that these corrections can significantly modify the Higgs mass prediction in models of low-energy supersymmetry breaking, for example, gauge mediation. Conversely, the Higgs mass measurement can be used to constrain the scale of mediation of supersymmetry breaking.

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Weak-scale supersymmetry provides a well motivated framework, both theoretically and phenomenologically, for extending the Standard Model of electroweak and strong interactions (SM). Consequently, supersymmetry has been the driving force in high-energy research for more than a decade. Its main attraction is the natural existence in supersymmetry of a fundamental Higgs boson. Furthermore, supersymmetry dictates that the quartic coupling in the SM Higgs potential,

$$V = -\mu^2 h h^\dagger + \frac{1}{2} \lambda |h h^\dagger|^2, \quad (1)$$

is given by a sum in quadrature of the electroweak hypercharge and $SU(2)$ couplings, g' and g , respectively,

$$\lambda = \frac{g'^2 + g^2}{4} \cos^2 2\beta. \quad (2)$$

The angle β , $\tan \beta \equiv \langle H_2 \rangle / \langle H_1 \rangle$, is a parameter of the (type II) two-Higgs-doublet model (2HDM) which generalizes the SM in its supersymmetric extension with H_1 (H_2) coupling to the b - (t -) quark. (The 2HDM is required by anomaly-cancellation constraints and holomorphicity). Here, we work in the decoupling limit of the 2HDM in which one physical Higgs doublet H is sufficiently heavy and decouples from electroweak symmetry breaking while a second SM-like Higgs doublet is roughly given (up to $\mathcal{O}(M_Z^2/M_H^2)$ corrections and a phase) by $h \simeq H_1 \cos \beta + H_2 \sin \beta$, and we conveniently defined

$$H_n = \begin{pmatrix} H_n^+ \\ (H_n^0 + iA_n^0)/\sqrt{2} \end{pmatrix}. \quad (3)$$

Hence, the mass of the physical SM-like Higgs boson $m_{h^0}^2 = \lambda v^2$ with $v = \langle h \rangle$ is now bounded at tree level,

$$m_{h^0}^2 \leq M_Z^2 \cos^2 2\beta, \quad (4)$$

by the Z -boson mass, $M_Z^2 = (1/4)(g'^2 + g^2)v^2$. This is all described in Ref. [1]. Note that h^0 parameterizes a flat direction of the 2HDM ($\beta \rightarrow \pi/4$), further suppressing its mass in the appropriate limit.

Supersymmetry, however, must be explicitly and softly broken with mass splitting between a fermion f and its scalar superpartner \tilde{f} , for example. Hence, new corrections to the quartic coupling arise quantum mechanically, the most important of which,

$$\delta\lambda_{soft} = \frac{3}{8\pi^2} y_t^4 \ln \frac{m_{\tilde{t}}^2}{m_t^2}, \quad (5)$$

arises from loops involving the top quark t and its superpartner the stop \tilde{t} , leading to $\mathcal{O}\left((m_t^4/M_Z^2) \ln(m_{\tilde{t}}^2/m_t^2)\right) \sim \mathcal{O}(100\%)$ radiative corrections to $m_{h^0}^2$ [2]. The importance of the corrections stems from the large coupling in the loop, the top-Yukawa coupling $y_t \sim 1$, and from the smallness of the tree-level mass Eq. (4). (The correction is maximized in the case of large $\tilde{t}_L - \tilde{t}_R$ mixing [3], which contributes additional terms to (5).) The upper bound (4) is now corrected by roughly a factor of a $\sqrt{2}$

$$m_{h^0} \leq 91 \text{ GeV} \rightarrow m_{h^0} \lesssim 130 \text{ GeV}, \quad (6)$$

with appropriately much smaller two-loop corrections. Hence, a strict upper bound on the Higgs boson mass exists even when soft supersymmetry breaking (SSB) effects are included, providing a strong prediction of supersymmetric extensions with minimal matter and gauge symmetries, commonly referred to hereafter as the MSSM. (More refined calculations can be done in particular MSSM realizations. For example, see Ref. [4].)

In non-minimal extensions involving, for example, an additional Abelian factor in the gauge group $\text{SM} \rightarrow \text{SM} \times U(1)$ or a SM singlet coupling to the two Higgs doublets $y_s S H_1 H_2$, new tree-level contributions to λ appear (from gauge D - and Yukawa F -terms, respectively), raising the upper bound (4) and modifying its β -dependence. This provides an important tool which discriminates between minimal and non-minimal realizations of supersymmetry. However, as long as perturbativity is assumed up to Planckian scales, one still has $m_{h^0} \lesssim 180 - 200 \text{ GeV}$ [5]. (One should bare in mind, however, that these limits are sensitive to the location of Landau poles which appear in the renormalization of many such models, and therefore do not provide as a strict limit as the MSSM limit Eq. (6). For examples, see Ref. [6].) Therefore, while the Higgs mass may discriminate between models (particularly if $\tan\beta$ is known independently), its lightness is largely model independent. A useful but rough approximation of the Higgs mass is as follows: Each contribution to the Higgs mass is at most $\sim 100\%$ of (4) $\sim M_Z$, and all contributions are summed in quadrature. For example, in models with both an extra $U(1)$ and additional SM singlet fields on finds, including loop corrections, $m_{h^0} \lesssim \sqrt{4}M_Z \sim 180 \text{ GeV}$ [7].

The SSB parameters such as $m_{\tilde{t}}^2$ above, carry mass dimensions and can contribute only logarithmically to the quartic couplings, and consequently, to the Higgs mass (once the vacuum expectation value (VEV) is fixed to its measured value). The Higgs mass is therefore more sensitive to the top mass (Yukawa coupling) than to the stop mass. This is the basis for the strong and model-independent results for the loop-corrected Higgs mass in the MSSM. Nevertheless, in general hard supersymmetry breaking (HSB) quartic couplings also arise (from non-renormalizable operators in the Kahler potential, for example). Assuming that the SSB parameters are characterized by a parameter $m_0 \sim 1 \text{ TeV}$ (i.e., $m_{\tilde{t}} \sim m_0$) then

$$\delta\lambda_{hard} = \tilde{\lambda}_h \frac{F^2}{M^4} \simeq \tilde{\lambda}_h (16\pi^2)^{2n} \left(\frac{m_0}{M}\right)^2, \quad (7)$$

where M is a dynamically determined scale parameterizing the communication of supersymmetry breaking to the SM sector, which is distinct from the supersymmetry breaking scale $\sqrt{F} \simeq (4\pi)^n \sqrt{m_0 M}$. Such operators were recently discussed in Ref. [8,9]. The exponent $2n$ is the loop order at which the mediation of supersymmetry breaking to the (quadratic) scalar potential occurs. (Non-perturbative dynmaics may lead to different relations that can be described instead by an effective value of n .) The coupling $\tilde{\lambda}_h$ is an unknown dimensionless coupling (for example, in the Kahler potential). As long as such quartic couplings are not arbitrary but are related to the source of the SSB parameters and are therefore described by (7), then they do not destabilize the scalar potential and do not introduce quadratic dependence on the ultra-violet cut-off scale, which is identified with M . Consider the one-loop contribution to a generic mass parameter in the scalar potential, which is given at tree level by the SSB scale $m^2|_{\text{tree}} = m_0^2$,

TABLE I. Frameworks for estimating $\delta\lambda_{hard}$. (Saturation of the lower bound on M is assumed.)

	n	$\tilde{\lambda}_h$	M	$\delta\lambda_{hard}$
TLM	0	~ 1	$\gtrsim m_0$	$(m_0/M)^2 \sim 1$
NPGM	1/2	~ 1	$\gtrsim 4\pi m_0$	$(4\pi m_0/M)^2 \sim 1$
MGM	1	$\lesssim 1/16\pi^2$	$\gtrsim 16\pi^2 m_0$	$(4\pi m_0/M)^2 \sim 1/16\pi^2$

$$\delta m^2 \sim \frac{\delta\lambda_{hard}}{16\pi^2} M^2 = \tilde{\lambda}_h (16\pi^2)^{2n-1} m_0^2 \lesssim m^2|_{\text{tree}}. \quad (8)$$

Stability of the scalar potential only constrains $\tilde{\lambda}_h \lesssim \min((1/16\pi^2)^{2n-1}, 1)$ (though calculability and predictability are diminished).

Such a hard coupling corrects (2) and as a result affects the tree level Higgs mass bound even in the MSSM. It introduces an explicit dependence of the Higgs mass on the supersymmetry mediation scale M , a dependence which is avoided in (5). In the case that supergravity interactions mediate supersymmetry breaking from some “hidden” sector (where supersymmetry is broken spontaneously) to the SM sector, one has $M = M_{Planck}$. The corrections are therefore negligible whether the mediation occurs at tree level ($n = 0$) or loop level ($n \geq 1$) and can be ignored for most purposes. (For exceptions, see Refs. [8,10].) In general, however, the scale of supersymmetry breaking is an arbitrary parameter and depends on the dynamics that mediate the SSB parameters. For example, it was shown recently that in the case of $N = 2$ supersymmetry one expects $M \sim 1 \text{ TeV}$ [9]. Also, in models with extra large dimensions the fundamental M_{Planck} scale can be as low as a few TeV, leading again to $M \sim 1 \text{ TeV}$. (For example, see Ref. [11].) A “TeV-type” mediation scale implies a similar supersymmetry breaking scale and provides an unconventional possibility. (For a discussion, see Ref. [9].) It may be further motivated by the observation that if the leading contribution to the cosmological constant (which vanishes in the supersymmetry limit) is $\sim M^8/M_{Planck}^4$ then observations suggest that $M \sim 1 \text{ TeV}$. (Other cosmological motivations for a TeV dynamical scale were discussed recently in Ref. [12].) If indeed $M \sim 1 \text{ TeV}$ then $\delta\lambda_{hard}$ given in (7) is $\mathcal{O}(1)$ (assuming tree-level mediation (TLM) and $\mathcal{O}(1)$ couplings $\tilde{\lambda}_h$ in the Kahler potential). The effects on the Higgs mass must be considered in this case.

A more familiar and surprising example is given by the (low-energy) gauge mediation (GM) framework [13]. In GM, SM gauge loops communicate between the SM fields and some messenger sectors, mediating the SSB potential. The Higgs sector and the related operators, however, are poorly understood in this framework [14] and therefore all allowed operators should be considered. In its minimal incarnation (MGM) $2n = 2$, and $M \sim 16\pi^2 m_0 \sim 100 \text{ TeV}$ parameterizes both the mediation and supersymmetry breaking scales. The constraint (8) corresponds to $\delta\lambda_{hard} \sim \tilde{\lambda}_h \lesssim 1/16\pi^2$ and the contribution of $\delta\lambda_{hard}$ to the Higgs mass could be comparable to the contribution of the supersymmetric coupling (2). A particularly interesting case is that of non-perturbative messenger dynamics (NPGM) in which case $n_{eff} = 1/2$, $M \sim 4\pi m_0 \sim 10 \text{ TeV}$ [15], and the constraint on $\tilde{\lambda}_h$ is relaxed to 1. Now $\delta\lambda_{hard} \lesssim 1$ terms could dominate the Higgs mass.

The different possibilities are summarized in the Table I. Next, we consider the β -dependence of the HSB contributions, which is different from that of all other terms.

In order to address the β -dependence, we revert temporarily to the 2HDM formalism of Haber and Hempfling [16]. The Higgs scalar potential can be written down as

$$\begin{aligned}
V = & m_{11}^2 H_1^\dagger H_1 + m_{22}^2 H_2^\dagger H_2 - [m_{12}^2 H_1^\dagger H_2 + \text{h.c.}] \\
& + \frac{1}{2} \lambda_1 (H_1^\dagger H_1)^2 + \frac{1}{2} \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 (H_1^\dagger H_2)(H_2^\dagger H_1) \\
& + \left\{ \frac{1}{2} \lambda_5 (H_1^\dagger H_2)^2 + [\lambda_6 (H_1^\dagger H_1) + \lambda_7 (H_2^\dagger H_2)] H_1^\dagger H_2 + \text{h.c.} \right\}.
\end{aligned} \tag{9}$$

Non-zero VEVs for the Higgs fields are obtained at the minimum of the scalar potential if $m_{11}^2 m_{22}^2 < |m_{12}|^4$. After electroweak symmetry breaking the five physical Higgs degrees of freedom (after diagonalizing the mass matrices) are two CP-even bosons H^0 and h^0 , with $m_{H^0} > m_{h^0}$, one CP-odd scalar A^0 and a charged Higgs H^\pm . The decoupling limit considered here is defined as $M_{A^0} \gg M_Z$: The heavy physical Higgs doublet $(H^+, (H^0 + iA^0)/\sqrt{2})^T$ decouples and the effective theory simply reduces to the Standard Model with one “light” physical Higgs boson h^0 , $m_{h^0}^2 = \lambda v^2$. The effective quartic coupling λ is related to the quartic couplings $\lambda_{1\dots 7}$ in the full 2HDM potential (9) via

$$\lambda = c_\beta^4 \lambda_1 + s_\beta^4 \lambda_2 + 2s_\beta^2 c_\beta^2 (\lambda_3 + \lambda_4 + \lambda_5) + 4c_\beta^3 s_\beta \lambda_6 + 4c_\beta s_\beta^3 \lambda_7, \tag{10}$$

where $s_\beta \equiv \sin \beta$ and $c_\beta \equiv \cos \beta$. Allowing additional hard supersymmetry breaking quartic terms besides the usual gauge (D -)terms and loop contributions, $\lambda_{1\dots 7}$ can be written out explicitly as

$$\lambda_{1,2} = \frac{1}{2}(g'^2 + g^2) + \delta\lambda_{soft\,1,2} + \delta\lambda_{hard\,1,2}, \tag{11}$$

$$\lambda_3 = -\frac{1}{4}(g'^2 - g^2) + \delta\lambda_{soft\,3} + \delta\lambda_{hard\,3}, \tag{12}$$

$$\lambda_4 = -\frac{1}{2}g^2 + \delta\lambda_{soft\,4} + \delta\lambda_{hard\,4}, \tag{13}$$

$$\lambda_{5,6,7} = \delta\lambda_{soft\,5,6,7} + \delta\lambda_{hard\,5,6,7}. \tag{14}$$

The leading SSB contribution $\delta\lambda_{soft\,i}$ were already summed in (5). The effect of the HSB contributions $\delta\lambda_{hard\,i}$ will be estimated below.

Substituting all the λ 's into Eq. (10), the squared Higgs mass $m_{h^0}^2$ reads

$$\begin{aligned}
m_{h^0}^2 = & \frac{1}{4}(g'^2 + g^2)v^2 \cos^2 2\beta + \delta m_{loop}^2 \\
& + (c_\beta^4 \delta\lambda_{hard\,1} + s_\beta^4 \delta\lambda_{hard\,2} + 2s_\beta^2 c_\beta^2 (\delta\lambda_{hard\,3} + \delta\lambda_{hard\,4} + \delta\lambda_{hard\,5}) \\
& + 4c_\beta^3 s_\beta \delta\lambda_{hard\,6} + 4c_\beta s_\beta^3 \delta\lambda_{hard\,7})v^2
\end{aligned} \tag{15}$$

$$\delta\lambda_{hard\,1=\dots=\delta\lambda_{hard\,7}=\delta\lambda_{hard}} M_Z^2 \cos^2 2\beta + \delta m_{loop}^2 + (c_\beta + s_\beta)^4 v^2 \delta\lambda_{hard}. \tag{16}$$

In (16), we have used the Z -boson mass and assumed for simplicity that all the $\delta\lambda_{hard}$'s are equal. Note that since no new particles or gauge interactions were introduced, (16) reduces to the familiar MSSM result (with only SSB terms) for $\delta\lambda_{hard\,1\dots 7} = 0$.

We are now in position to evaluate the HSB contributions to the Higgs mass for an arbitrary M (and n). In Figs. 1 and 2, we show the dependence of the mass of the SM-like Higgs boson on the effective mediation scale M_* , including both the loop corrections (upper-bound values adopted from Ref. [17] and incorporated numerically) and the HSB contributions, for $\tan \beta = 1.6$ and 30 respectively. The effective scale is defined as $M_* \equiv (M/(4\pi)^{2n} \sqrt{\tilde{\lambda}_h})(\text{TeV}/m_0)$. The HSB contributions decouple for $M_* \gg m_0$, and the results reduce to the MSSM limit with only SSB (e.g., supergravity mediation). However, for

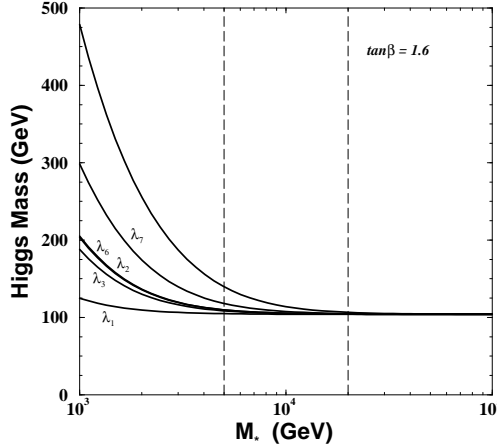


FIG. 1. The SM-like Higgs boson mass for $\tan \beta = 1.6$ is shown as a function of the effective mediation scale M_* . The contribution of each individual $\delta\lambda_{hard i}$ HSB coupling (when added to the result with only SSB, as explained in the text) is shown (with the label λ_i), as well as their sum (upper curve, assuming that all HSB couplings are equal). The M_* range in the MGM case is indicated (dashed lines) for reference. For sufficiently large values of M_* all curves converge to the MSSM with only SSB upper-bound value.

smaller values of M_* the Higgs mass is dramatically enhanced. For $M=1$ TeV and TLM or $M=4\pi$ TeV and NPGM, both of which correspond to $M_* \simeq 1$ TeV, the Higgs mass could be as heavy as 475 GeV for $\tan \beta = 1.6$ and 290 GeV for $\tan \beta = 30$. For small values of $\tan \beta$, the $\delta\lambda_{hard 7}$ term gives the dominant contribution (since it is enhanced by a factor of four), while for large $\tan \beta$ the $\delta\lambda_{hard 2}$ term dominates.

In the MGM case $\tilde{\lambda}_{hard} \lesssim 1/16\pi^2$ so that $M_* \sim 4\pi$ TeV (unlike the NPGM where $M_* \sim 1$ TeV). Given the many uncertainties (e.g., the messenger quantum numbers and multiplicity and \sqrt{F}/M [13]) we identify the MGM with a M_* -range which corresponds to a factor of two uncertainty in the hard coupling. HSB effects are now more moderate but can increase the Higgs mass by 40 (10) GeV for $\tan \beta = 1.6$ (30) (in comparison to the MSSM with only SSB.) Although the increase in the Higgs mass in this case is not as large as in the TLM and NPGM cases, it is of the same order of magnitude as or larger than the MSSM two-loop corrections [17], setting the uncertainty range on any such calculation. Clearly, within the MSSM the Higgs mass could discriminate between the MGM and NPDM and help to better understand the origin of the supersymmetry breaking.

In Fig. 3, m_{h^0} dependence on $\tan \beta$ for fixed values of M_* is shown. The $\tan \beta$ dependence is from the tree-level mass and from the HSB corrections, while the loop corrections to $m_{h^0}^2$ are fixed, for simplicity, at 9200 GeV^2 [17]. The upper curve effectively corresponds to $\delta\lambda_{hard} \simeq 1$. The HSB contribution dominates the Higgs mass and m_{h^0} decreases with increasing $\tan \beta$. As shown above, m_{h^0} could be in the range of 300 – 500 GeV, dramatically departing from all previous MSSM calculations which ignored HSB terms even in the case of low-energy supersymmetry breaking. The lower two curves illustrate the range of the corrections in the MGM, where the tree-level and the HSB contributions compete. The $\cos 2\beta$ dependence of the tree-level term dominates the β -dependence of these two curves.

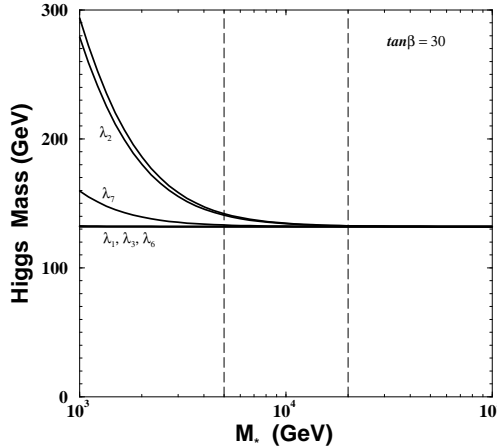


FIG. 2. The same as in Fig. 1 except for $\tan \beta = 30$.

Following the Higgs boson discovery, it should be possible to extract information on the mediation scale M . In fact, some limits can already be extracted. Consider the upper bound on the Higgs mass derived from a fit to electroweak precision data: $m_h^0 < 215$ GeV at 95% confidence level [18]. (Such fits are valid in the decoupling limit discussed here.) A lower bound on the scale M in GM could be obtained from

$$m_Z^2 \cos^2 2\beta + \delta m_{loop}^2 + (c_\beta + s_\beta)^4 v^2 \left(\frac{4\pi m_0}{M} \right)^2 \leq (215 \text{ GeV})^2, \quad (17)$$

assuming equal $\delta\lambda_{hard}$'s. For $\beta = 1.6$, it gives $M \geq 31$ TeV while for $\tan \beta = 30$ the lower bound is $M \geq 19$ TeV. Once m_{h^0} is measured, more stringent bounds on M could be set.

In conclusion, we illustrated that the scale of the mediation of supersymmetry breaking explicitly appears in the MSSM prediction of the Higgs mass, and with a distinct β -dependence. (It would also appear in any expression for the Higgs mass derived in extended models, which correspond to a straightforward generalization of our discussion.) In turn, it could lead in certain cases to a much heavier MSSM Higgs boson than usually anticipated. It could also distinguish models, e.g., supergravity mediation from other low-energy mediation and weakly from strongly interacting messenger sectors. Given our ignorance of the (Kahler potential and) HSB terms, such effects can serve for setting the uncertainty on any Higgs mass calculations and can be used to qualitatively constrain the scale of mediation of supersymmetry breaking from the hidden to the SM sector.

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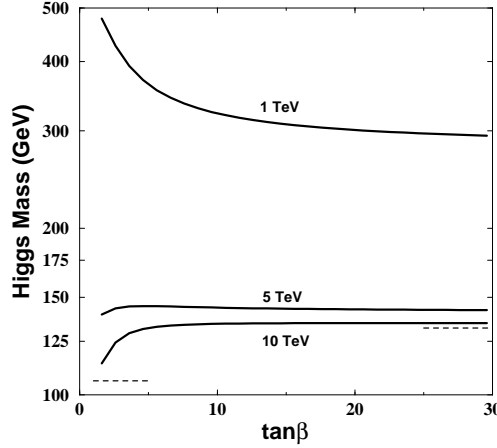


FIG. 3. The light Higgs boson mass (note the logarithmic scale) is shown as a function of $\tan\beta$ for $M_* = 1, 5, 10$ TeV (assuming equal HSB couplings). The upper bound when considering only SSB ($M_* \rightarrow \infty$) is indicated for comparison (dashed lines) for $\tan\beta = 1.6$ (left) and 30 (right).

REFERENCES

- [1] J.F. Gunion *et al.*, *The Higgs Hunter Guide* (Addison Wesley, Redwood City, CA, 1990).
- [2] H.E. Haber and R. Hempfling, Phys. Rev. Lett. **66**, 1815 (1991); Y. Okada, M. Yamaguchi and T. Yanagida, Prog. Theor. Phys. **85**, 1 (1991); J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. **B257**, 83 (1991).
- [3] P. Langacker and N. Polonsky, Phys. Rev. **D50**, 2199 (1994); J.A. Casas *et al.*, Nucl. Phys. **B436**, 3 (1995).
- [4] S. Su, Nucl. Phys. **B573**, 87 (2000).
- [5] J.R. Espinosa and M. Quiros, Phys. Rev. Lett. **81**, 516 (1998).
- [6] M. Masip, R. Munoz-Tapia and A. Pomarol, Phys. Rev. **D57**, 5340 (1998).
- [7] P. Langacker, N. Polonsky and J. Wang, Phys. Rev. **D60**, 115005 (1999).
- [8] S.P. Martin, Phys. Rev. **D61**, 035004 (2000).
- [9] N. Polonsky and S. Su, MIT-CTP-2987, hep-ph/0006174.
- [10] H.P. Nilles and N. Polonsky, Phys. Lett. **B412**, 69 (1997); K. Choi, J.S. Lee and C. Munoz, Phys. Rev. Lett. **80**, 3686 (1998); M.K. Gaillard and B. Nelson, hep-th/0004170.
- [11] P. Delgado, A. Pomarol and M. Quiros, Phys. Rev. **D60**, 095008 (1999).
- [12] N. Arkani-Hamed *et al.*, astro-ph/0005111.
- [13] For a review, see G.F. Giudice and R. Rattazzi, Phys. Rept. **322**, 419 (1999).
- [14] For a review, see N. Polonsky, hep-ph/9911329.
- [15] A.G. Cohen, D.B. Kaplan and A.E. Nelson, Phys. Lett. **B412**, 301 (1997).
- [16] H.E. Haber and R. Hempfling, Phys. Rev. **D48**, 4280 (1993).
- [17] M. Carena *et al.*, Nucl. Phys. **B580**, 29 (2000).
- [18] Particle Data Group, C. Groom *et al.*, Eur. Phys. J. **C15** 1 (2000).